Intl. J. Engg. Sci. Adv. Research 2020 September; 6(3): 8-13 ISSN NO: 2395-0730

Review of Geometrical Tolerance Analysis

Gyanendra Kumar Yadav [1] and Pankaj Kumar Yadav [2]
Assistant Professor[1,2]
Department of mechanical Engineering,
School of Management Science, Lucknow

Abstract- **A Tolerance specification is an important part of mechanical design tolerancing decisions can profoundly impact the quality and cost of the mechanism. To evaluate the impact of tolerance on mechanism quality, designers need to simulate the influences of tolerances with respect to the functional requirements. Design tolerances strongly influence the functional performance and manufacturing cost of a mechanical product. Tighter tolerances normally produce superior components, better performing mechanical systems and good assimilability with assured exchangeability at the assembly line. However, unnecessarily tight tolerances lead to excessive manufacturing costs for a given application. The balancing of performance and manufacturing cost through identification of optimal design tolerances is a major concern in modern design.**

Computer-aided (or software-based) tolerance synthesis and alternative manufacturing process selection programs allow a designer to verify the relations between all design tolerances to produce a consistent and feasible design.

Keyword-Tolerancing, Interchangebility, Geometric Dimensional Tolerance

I. INTRODUCTION

Life and performance of mechanical product depend on many factors; tolerance is one of the major parameter. Good tolerance design gives intricate mechanical assemblies containing many parts to assemble and work together in a smoother and desirable way to fulfil the design objectives. Due to mass production tolerance play a vital role in design phase Figure (1). With increase in technology and requirements of more reliable geometry need of tighten tolerance increase, thus the cost and the required precision of assemblies increases. Therefore there is need for increase in research for tolerance design in order to enable high-precision assemblies for manufacturing at lower costs.

To improve the tolerance limit in a manufacturing industry, there is very strong need for tolerance analysis to estimate the probability expressed in ppm (defected product per million) with high-precision computed at lower cost. The designer needs tolerance analysis

- To increase product performance
- To minimize the manufacturing cost
- To decrease scraps in production process

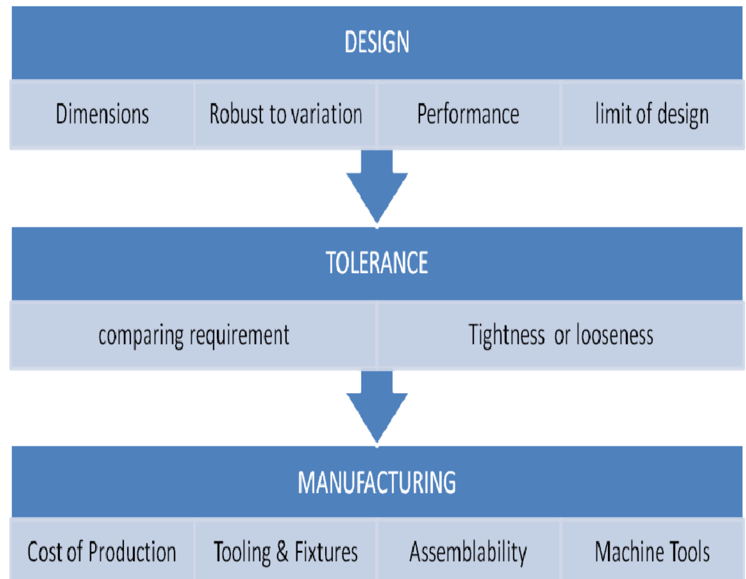

Figure 1

II. LITERATURE REVIEW

Too much research have been done in the field of traditional tolerance. Traditional tolerance methods are great for dimensioning limits and tolerance of different size with good capacity. But these methods are not working for cater precisely for, profile, run out, locations, form and orientation features [1] [2] [3]. Geometric Dimensioning and tolerancing is used for locations, profiles, run out, form and orientation features. Actually GDT can be categorized mainly into two types of tolerance schemes i.e. dimensional and geometric. Dimensional tolerance consists of identifying a set of dimension and assigning limits to the dimensions and defines a range of values [4]. The different methods of tolerance allocation are found mean shift models, the combination of the basic approaches and tolerance synthesis approaches for tolerance stack up i.e. the worst case and the root sum square approach [5]. Introduction of statistical based approach to tolerance allocation and in this approach, a systematic analysis for estimating process capability levels at the design stage is used in conjunction with statistical methods for the optimization of tolerances in assembly stack up and methods are demonstrated for the tolerance allocation of minimum production cost can be extended to include process selection from a set of alternate processes [6]. The stack up of geometrical tolerances using generic capsule method present an elegant approach by using the 'Quickie' technique towards tolerance stack up analysis for geometrical tolerances. A straightforward graphical approach known as the "Catena" method for tolerance stack up, which involve the geometric characteristics in form control – flatness, straightness, circularity and cylindricity [7]. The development of an extension of computerized trace method to determine the relationship between geometrical tolerances and manufacturing dimensions and tolerances [8]. This method minimizes the cost of scraps is the objective function which is a function of manufacturing tolerances. Requirements of design sizes, geometrical tolerances (both form and position) and machining allowances are expressed mathematically as constraints for the optimization. General new methodology using intelligent algorithms for simultaneous optimal selection of design and manufacturing tolerances with alternative manufacturing process selection [8]. An original method that enables to solve problems for the case of serial assembly (stacking) without clearances this method is based on the use of influence coefficients to obtain the relationship between the functional tolerance and the tolerances associated with the geometry of the mechanism's interface surfaces [9] [10]. Sahani et al [11] presented review of different techniques for stack up for flatness geometrical tolerances

The two phases of synthesis are as follows. The first phase obtains the equations that relate geometric deviations to variations in functional conditions, which is refered as tolerancing equations. The second phase uses a statistical approach. The worst-case method considers the inequalities that are related to geometric deviations which are bounded by tolerance. Statistical method includes statistical data which are related to deviation variables that are assumed as known, they are helpful to get statistics on variables which usually describe the functional conditions.

The tolerance analysis of mechanical assemblies where assembly performance is subject to internal or external forces. The theoretical nature of manufacturing processes proceeds to deviation in desired product which directly or indirectly affects the performance and cost of manufactured products. Entertaining the effects of manufacturing deviation at initial stage of product design is primary way for achieving the competitive quality, cost and advancement of technology in the future. Manufacturing deviation is computed by tolerances and manufacturing process tendency. Tolerances indicate the permissible variation around a apparent parametric value. The geometry, position, orientation and size of toleranced characteristic are described according to Geometric Dimensioning and Tolerance (GD&T) standards. [12]

The tolerance decisions can extremely affect the quality and cost of the mechanism. To calculate the effect of tolerance on product quality, designers need to resemble the importance of tolerances with respect to the functional requirements. This paper proposes a mathematical formation of tolerance analysis which consolidate the understanding of quantifier: ''For all acceptable variation (variations those are inside), there exists a gap configuration such as the assembly requirements and the behaviour constraints are verified'' & ''For all acceptable variation (variations those are inside tolerances), and for all permissible gap configurations, the assembly and functional requirements and the behaviour constraints are verified''. The quantifiers provide an absolute expression of the condition corresponding to a geometrical product requirement. This provides a large area for research in tolerance analysis. To solve the mechanical problem, and method established on optimization is proposed. Monte Carlo simulation is accomplished for the statistical analysis. The proposed approach is also tested on an over-constrained mechanism. [13]

Tolerances specify the ranges of permissible feasibility so that a manufactured part will be acceptable for assembly or not. Standardization defines classes of tolerances and to ensure proper interchange ability among product assembly. The purpose of this research is devoted for creating a mathematical model of geometric tolerances. The results from earlier effort show that the proposed math model is compatible with the standards, and that it provides three-dimensional relations for assemblies. At the initial stage, the method of the new model is to represent each tolerance-zone for a plane or a line as a imaginary point-space, map, and to combine these for assemblies of units and at the final stage, the model inter-relates all reference frames of an assembly using degrees of freedom. The project includes the implementation of a tolerance analysis system for comparison of the new model with existing software.[14]

Existing methods for analysis of tolerances today are based strongly on *ad-hoc* conventions from engineering practice and less on mathematical principles. Therefore full three-dimensional analyses of tolerances in assemblies are not done yet, and current design software are only partially compatible with existing standards. The result of this research will provide the means to complete full three-dimensional statistical analyses of tolerances and thus it will enhance the quality and minimize cost. [28]

The productivity and industrial product quality improvements involve a rational tolerance process to be applied at initial product design. Firstly functional conditions are defined; an optimal specification for each part of a mechanical system is to be developed. Despite various researches in this field, the problem is still unsolved and it is under development phase. It may be categorized into two stages: development of specifications based on standards, or qualitative synthesis, and calculation of tolerances. To the scope of these two types of problems are related, they propose to address them in parallel. Mansuy, M., et al presents an original method that enables us to solve these two problems for the case of serial assembly (stacking) without clearances. That technique is based on the use of influence coefficients to obtain the relationship between the functional tolerance and the tolerances associated with the geometry of the mechanism's interface surfaces. They describe a calculation algorithm that helps obtain influence coefficients purely from the assembly's geometric definition and shows that under there working hypothesis, this relationship is piecewise linear. [15]

Related works: The significance amount of research has been dedicated for the development of tolerance analysis. Tolerance analysis is interested in the verification of the functional requirements after tolerance has been described on every element. There are three main issues in tolerance analysis.

(a) The models for describing the geometrical variation and gaps: the variation of the real entity from the ideal entity in 3D can be described in any one of the following way:

- With the help of the vectors or vectorial tolerancing
- With the torsors of the small displacements
- From matrices
- From kinematic formulation or a kinematic method
- From stream of variations (SOVA)

(b) A mathematical method for manipulating the system behaviour with variation,

(c) The development of the solving techniques and method analysis, such as worst-case and statistical analysis [16]

Worst-case analysis considers the worst possible combination of each variation and also considers the functional behaviour. Therefore, worst-case tolerance technique lead to excessively tight part tolerances and hence high production costs [17]. Statistical tolerancing on other hand works by setting the tolerances so as to ensure desired yield. By permitting a small fraction of assemblies to not assemble or function as required, an increase in tolerances for individual dimensions may be obtained, and in turn, manufacturing costs may be reduced significantly [25].

The Tolerance analysis methods are classified into two different categories based on the type of accumulation input: displacement accumulation or tolerance accumulation. The aim of displacement accumulation is to simulate the influences of deviations on the geometrical behaviour of the mechanism. Usually, tolerance analysis uses a relationship of the form [18]:

$Y = f(X)$, (1)

where Y is the response (characteristic such as gap or functional characteristics) of the mechanism and $X = \{x1, x2, . . . , xn\}$ are the values of some characteristics (such as situation deviations or/and intrinsic deviations) of the individual parts or subassemblies making up the mechanism. The function f is the mechanism response function which represents the deviation accumulation.

For statistical tolerance analysis, the input variables $X = \{x1, x2, xn\}$ are continuous random variables which enable to represent part deviations. In general, they could be mutually dependent. A variety of methods and techniques Linear Propagation (Root Sum of Squares), Non-linear propagation (Extended Taylor series), Numerical integration (Quadrature technique), and Monte Carlo simulation are available for estimation of the probability distribution of Y and the probability of the respect to the geometrical requirement [18].

The geometrical tolerances or the dimensioning tolerances are represented by deviation space [21–24], T-Map or specification hull [19-20]

III. MODEL FOR TOLERANCE ANALYSIS

Tolerance analysis contains propagation and representation of tolerance. Tolerance analysis involves the whole process of the product, contain manufacturing, product process planning, product design, inspection and testing, but actually the aim may be different in each case.

Table 1 List Tolerance representation and propagation method.

T-Representation	T-Propagation
Variational Geometry	Linearization Method
Variational Class	Quadrature
Virtual Boundary	Reliability Index
Feasibility Space	Taguchi Method
Vectorial Approach	Monte Carlo Simulations
Degree of freedom (DOF)	Direct linearization method
Tolerance Map	Jacobian Matrix
Topologically and technologically related surface	Variational method
Infinitesimal matrix	State space
Small displacemenal torsor Proportional assembly clearance volume	Kinematic formulation
Small displacemenal torsor	Network of zone and datums
Virtual joints	System moments

The methods which we are going to discuss are Tolerance Map, Direct Linearization Method, Unified Jacobian Torsor Model, Matrix model and Monte Carlo Simulations.

(a) **Tolerance Map-** Tolerance Map shows all probable changes in the shape, size, position, and orientation for a desired assembly. T-Map method can represent all of the tolerance and their interaction with the mathematical equations and operation that may be difficult.

(b) **Direct Linearization Method-** Direct linearization method base on the first order Taylor's series expansion of vector loop-based assembly models which use vectors to represent either component dimensions or assembly dimensions.

(c) **Unified Jacobean Torsor Model**- The Unified Jacobean Torsor Model combines the advantage of the torsor model which is suitable for tolerance representation and the Jacobean matrix which is suitable for tolerance propagation.

(d) **Matrix Model-** The matrix model a homogeneous matrix that are traditional and brief but comprehensive.[17]

3.1 Geometric tolerance analysis

The geometric tolerance can be categorized into four main category, i.e., form tolerance orientation tolerance location tolerance and run out tolerance, according to GB/T 1182-2008[26]:

(1). A form tolerance defines how much an actual surface or feature is permitted to vary from the desired form implied by the drawing. It consists of flatness, circularity, straightness, profile of a line and profile of a surface which is represented as shown in figure (2,3).

(2). An orientation tolerance defines how much an actual surface or feature is permitted to vary relative to a datum. It consists of perpendicularity, parallelism, angularity, profile of a line and profile of a surface as shown in figure (4,5).

(3). A location tolerance defines how much an actual size feature is permitted to vary from the perfect location implied by the drawing as related to a datum, or other features. This category includes position, concentricity, symmetricity profile of a line and profile of a surface.

(4). A run out tolerance define how much an actual surface or feature is permitted to vary from the desired form implied by the drawing during full (360°) rotation of the part on a datum axis. A run out can be either a circular run out or a total run out. Tolerance region is the space limited by one or several geometrically perfect lines or surfaces, and characterized by a linear dimension, called a tolerance.

According to the specification of the tolerance and the way in which it is to be dimensioned, the tolerance zone is one of the following:

(1). Area fluctuation of a circle;
(2). Gap between two concentric circles;
(3). Gap between two equidistant lines or two parallel straight lines;
(4). Area fluctuation of a cylinder;
(5). Gap between two coaxial cylinders
(6). Gap between two equidistant surfaces or two parallel planes;
(7). Area fluctuation of a sphere.

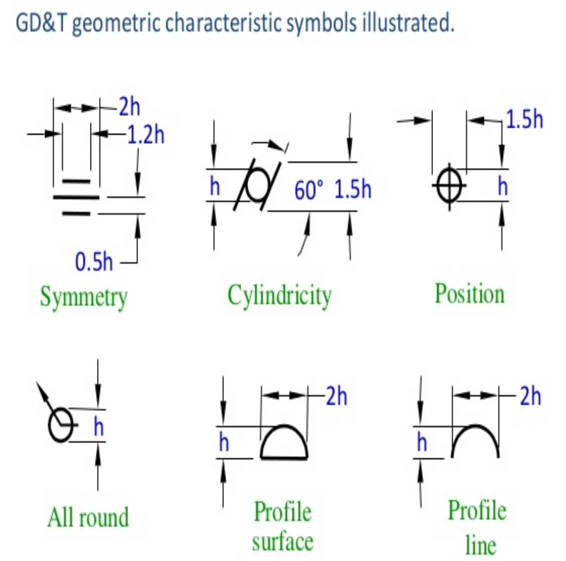

Figure 2 Geometric Dimensional tolerance symbols

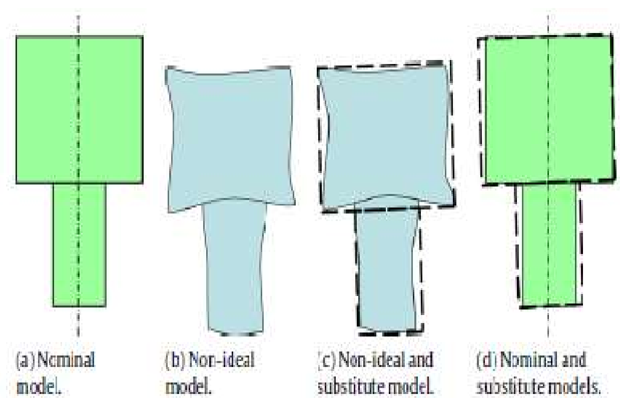

Figure 3 Ideal and non-ideal geometry

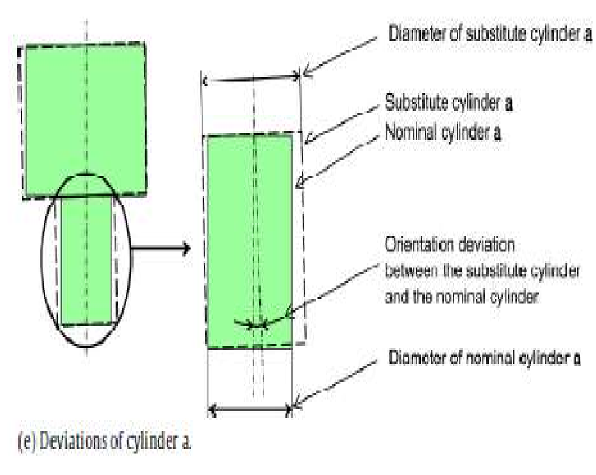

Figure 4 Deviation of axis

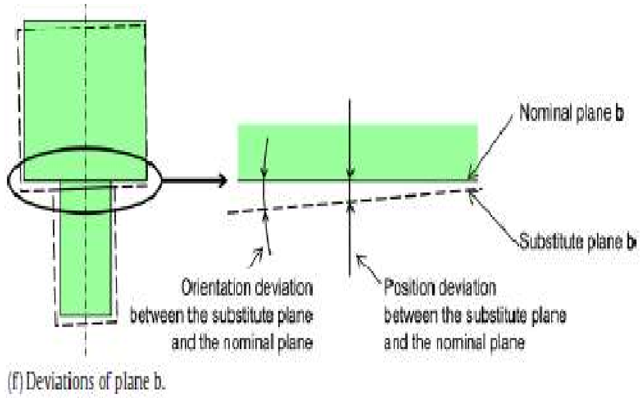

Figure 5 Deviation of plane

3.2 The Effect of Geometric Tolerance on the Tolerance Analysis

There are three major issues of deviation in a mechanical assembly .Two of them are due to the natural abnormality in manufacturing processes and the third is in the form of assembly process and proceeding. These three sources are

1) Dimensional fluctuation
2) Geometric deviation and
3) Fluctuation due to kinematic arrangement at assembly time.

The geometric tolerance can be considered as dimension tolerance whose nominal value is zero and added to the related contact surfaces. And its direction is mainly resolved by the kinematic joints and the geometric behaviour.

The effect of the geometric tolerances correlated with each of the joins may result in conventional variation or rotational variation. Figure (6) illustrates how a flatness tolerance of the lower plane can affect two mating parts differently when viewed in 2-D. The cylinder on the left represents a conventional variation while the block on the right exhibits the rotational variation, due to the same geometric variation. The conventional variation for the cylinder in Figure (6) is related to the flatness tolerance, while the rotational variation for the block is determined by not only the flatness tolerance but also by the contact length of the block, in this case, the horizontal dimension of the block.

$$\Delta\alpha = \pm \frac{1}{2} T \qquad (1)$$

$$\Delta\beta = \pm\, tan^{-1} (L/T) \qquad (2)$$

Where T is the flatness tolerance, L is the contact length of the block, $\Delta\ \alpha$ is the conventional variation caused by the flatness in a planar joint, and $\Delta\ \beta$ is the rotational variation resulted from the flatness in a planar joint and the contact length of the block.[29]

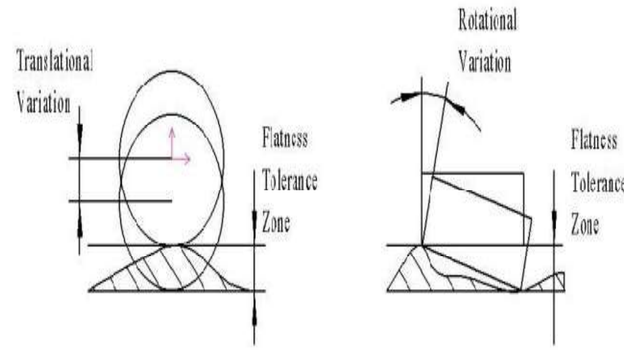

Figure 6 Effect of undesirable geometry

Research has been dedicated for the advancement of tolerance analysis. It may be either worst-case analysis or statistical analysis.

Worst-case analysis (deterministic or tight fit tolerance analysis) involves establishing the dimensions and geometric tolerances such that any possible combination produces a functional assembly, i.e. have 100% probability of being assembly in first attempt compatibly. It considers the worst possible combinations of individual tolerance and examines the functional behaviour. Therefore worst-case tolerance analysis can lead to excessively tight part tolerances and thus very high production costs.

Statistical tolerance analysis is a more realistic and feasible way of looking at tolerance and works on setting the tolerance to ensure the expected yield. By allowing a small amount of assemblies not to function as required, an increase in tolerance for individual dimensions may be obtained, and in return, manufacturing costs may be decreased notably. Statistical tolerance analysis enumerates the probability that the object can be assembled and will perform under a given unique tolerance [27].

IV. CONCLUSION

The main aim of this work is to conduct research on and then verify mathematics of geometric variational model for different categories of dimensions and tolerances accurate with tolerance standards and compatible with dimensional solid models.

The mathematical model will provide improvements in the designer's ability to determine the following problems that are important in design.

a) The contribution of dimensional features for the distinct assemblies' position.
b) Ability to assign the permissible net change in the mating parts between all the contributing dimensions and geometric variations

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