Investigation of Signal Processing Techniques for the Analysis of Impulsive Signals

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***Abstract*—This paper reports the effect of different signal processing techniques on the analysis of an impulsive signal. Different signal processing techniques such as the Statistical, Fourier, Short-time Fourier and Wavelet transform techniques are studied on a free decay signal. The original signal is transformed using the transforms, parameters are estimated and the signals are reconstructed. The error is estimated between the parameters of reconstructed signal and original signal. The correlation coefficient is established between the reconstructed signal with the original signal. The correlation coefficients and errors are used as evaluation criteria for the assessment of the transform. The results indicate the different transforms have different errors, coefficients and thus provides the degree of superiority to assess the transform.**

***Index Terms*—** *Statistical, Fourier, Short-time Fourier and Wavelet transform.*

I. Introduction

IMPULSIVE signals appear in every phenomena, especially during the start or shutdown of a machinery, collision or impact of bodies. The transient signals are short lived; die out quickly and gives rigorous information regarding the natural characteristics of the system. While the harmonic signals act under the continuous forcing function and depend on the amplitude and frequency of the excitation. A transform decomposes the signal into its constituent notes which can be used to compress the data and reconstruct the original signal. A simplest transform is the Fourier transform, this is applied to the analysis of harmonic signals and the transform is derived from the Fourier series. The Fourier series states that some functions could be written as an infinite sum of harmonics. The Fourier transforms yield significant error when applied to the analysis of transient signals and looses the time information. The short-time Fourier transform has a fixed resolution and makes the compromise between the time and frequency, the recent Wavelet transform has varied the resolution depending on the signals. The wavelet theory says a signal can be decomposed into an infinite sum of wavelets, which can be scaled or translated from a mother wavelet. The objective of this study is to investigate these methods in studying the impulsive signals.

The Section 2 of this paper reviews the mathematical preliminaries of these transforms, a transformation of a discrete signal is carried out and the results are discussed in the later section. The last section of this paper presents conclusions.

II. Signal Analysis Methods

The signal is processed to extract the salient features, in statistical analysis various first order and higher order moments are selected based on the signals. Most often for machinery signals mean, peak, rms and kurtosis feature are used. These features can be further analyzed using the statical classifiers such as principal component analysis, discrement analysis etc.,

1. Statistical Parameters

Absolute Mean: Expected value of the signal is

$$\bar{y} = E(y_i) = \frac{1}{n}\sum_{i=0}^{n} y_i \tag{1}$$

Maximum Peak: Highest amplitude is the signal is

$$y_p = max(y_i) \tag{2}$$

Root Mean Square: Measure of the power content in signal, usually used to track the overall noise level.

$$y_{RMS} = \sqrt{\frac{1}{n}\sum(y_i)^2} \tag{3}$$

Kurtosis: Measure of peak-ness or flatness of a distribution

$$Kur = \frac{1}{{y_{RMS}}^4} E(y_i - \bar{y})^4 \tag{4}$$

2. Fourier Transform (FT):

$$F(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\cdot\omega t}\, dt \tag{5}$$

3. Short Time FT (STFT):

$$STFT_f^w(\tau,\omega) = \int_{-\infty}^{\infty} y(t) \cdot w^*(t-\tau) \cdot e^{-j\cdot\omega t}\, dt \tag{6}$$

4. Wavelet Transform (WT):

$$CWT_f^{\psi}(\tau,s) = \int_{-\infty}^{\infty} y(t)\,\psi_{\tau,s}^*(t)dt\,; \quad s>0, \tau \in \mathcal{R} \qquad (7)$$

Where

$\psi_{\tau,s}^*(t) = \frac{1}{\sqrt{s}}\psi^*\left(\frac{t-\tau}{s}\right)$ is the complex conjugate of base function or mother wavelet.

The parameters of the signal such as amplitude, frequency and decay rate can be extracted from these transforms. The extracted information from each transform can be compared with the original signal and error in each transform is used as the criteria for assessing the transform.

III. Simulation Evaluation

In the simulation evaluation a simple impulsive signal with known parameters is constructed, then the signal is transformed using Fourier, Short time Fourier and Wavelet transforms. The constituent notes from the transform are extracted and the signal is reconstructed. The correlation is estimated between the reconstructed signals with the original signal.

$$y(t) = Y_0 e^{-\zeta\omega t}\,\sin(\omega t) \qquad (8)$$

The amplitude 10 units, modal damping 0.02, frequency 7.9

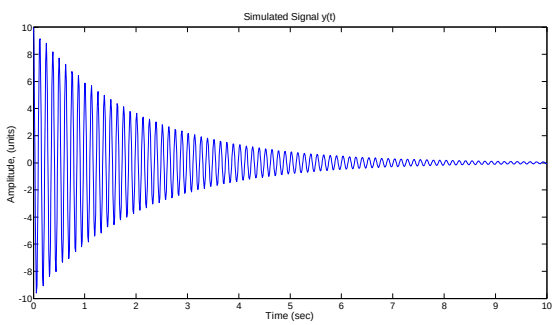

Fig. 1. Simulated impulsive signal.

Hz are considered for duration 10 sec in constructing the signal as shown in Fig. 1.

The statistical quanaties indicate the time and frequency

TABLE I Stastical Properties of the Impulse Signal

Symbol	Quantity	Conversion from Gaussian and CGS EMU to SI [a]
μ_y	Absolute Mean	0.0051Units
y_p	Maximum Peak	10.0 Units
y_{RMS}	Root Mean Square	2.2462 Units
Kur	Kurtosis	7.5593 Dimension less

independent stastic process. The table 1 gives the Descriptive statistics of the signal.

It can be seen that the only peak amplitude gives meaningful result, other parameters are less effective. The type and kind of occurrence also not captured so this method is not effective in reconstructing hence this method is suitable for random and study signals.

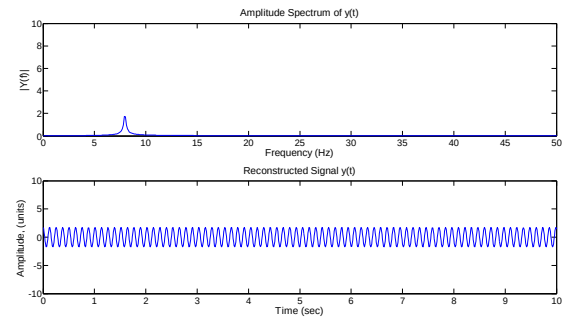

Fig. 2. Fourier transform and reconstructed signal.

A Fourier transform of the above signal is carried out using a single sided FFT algorithm. The Fig 2 (a) shows the FFT transform and Fig. 2 (b) shows its reconstruction. The peak amplitude and corresponding frequency information are extracted from the plot and the signal is reconstructed

It can be seen that the time information has lost and the amplitude has significantly lowered, since FFT translate in to steady harmonic components so the reconstructed signal going to exist for complete duration. The amplitude 1.7318 units, modal damping 0.0, frequency 7.91 Hz are identified from this

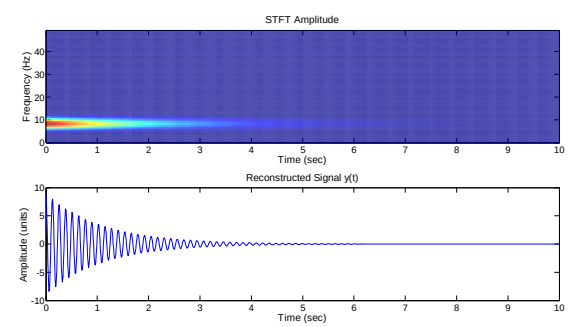

Fig. 3. Short time Fourier transform and reconstructed signal.

transformation. It indicates the frequency is well estimated while the amplitude is poorly estimated and it is unable to estimate the damping.

The Fig. 3 shows the short time Fourier transform of the signal. The signal is transformed with the frequency bin width 0.7813 Hz and window resolution 0.1.

The amplitude 9.746 units, modal damping 0.017, frequency 7.813 Hz are identified from this transformation. The Fig. 3 (b) shows its reconstructed signal. It indicates the frequency is as well as damping is well estimated as compared to the

Fourier transform.

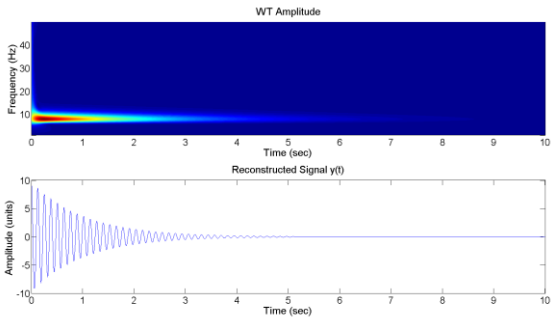

Fig. 4. Wavelet transform and reconstructed signal.

The Fig. 4 shows the Wavelet Transform of the signal. The Morlet wavelet is closed for the analysis. The signal is

TABLE II COMPARISON OF TRANSFORMS

Parameter	Simulation	Statistical	FT	STFT	WT
Amplitude, units	10	10 (0)	1.732 (-22.68)	9.746 (-2.54)	9.835 (-1.65)
Frequency, Hz	7.9	-	7.91 (0.127)	7.813 (-1.1)	7.88 (0.25)
Damping, -	0.02	-	-	0.017 (-15)	0.197 (-1.5)

transformed with the frequency bin width is 128 voices for every dyadic interval and window resolution 0.1. The amplitude 9.835 units, modal damping 0.0197, frequency 7.88

Hz are identified from this transformation. The Fig. 4 (b) shows its reconstructed signal. It indicates the frequency is as well as damping is well estimated as compared to the STFT and Fourier transform.

The Table. 2 shows the comparison of different results with % error in the respective transform shown in closed brackets.

The results indicate there the WT is able to predict the simulation accurately with error around 1% while statistical and FT are unable to estimate certain parameters..

IV. CONCLUSION

Different signal processing techniques such as the Statistical, Fourier, Short-time Fourier and Wavelet transform techniques are investigated on a free decay signal. The wavelet based signal processing shows an efficient way of estimating. The estimated parameters are close to the parameters used for constructing the signals. The comparison shows the wavelet transform is the best way for analysis the impulsive signals.

REFERENCES

[1] S.Mallat, “A Wavelet Tour of Signal Processing”, 2nd ed., Academic Press,1999.2

[2] Staszewski, W. J., “Identification of damping in MDOF systems using time-scale decomposition”, Journal of sound and vibration 203.2 (1997): 283-305.

[3] Slavič, Janko, Igor Simonovski, and Miha Boltežar., “Damping identification using a continuous wavelet transform: application to real data”, Journal of Sound and Vibration 262.2 (2003): 291-307.

[4] Jaideva C. Goswami, Andrew K. Chan, “Fundamentals of Wavelets: Theory, Algorithms, and Applications”, 2nd Edition, 2011 John Wiley & Sons, Inc.